

THE TOWERS OF HANOI-6

If you want to know some more...

Voyage to the Country of Knowledge

For this voyage, we will use a code:

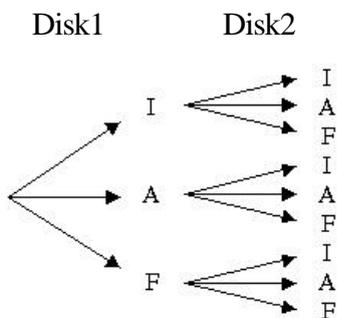
Disk 1: biggest disk	I: initial tower
Disk 2: disk following to 1	A: auxiliar tower or intermediate
Disk 3: disk following to 2	F: final tower
I_1F_2 represents the following situation: disk 1 is in the initial tower and disk 2 in the final one.	

... and two powerful matematical tools:

tree diagrams and graphs.

Let's start the voyage by analyzing the possible positions of 2 disks¹:

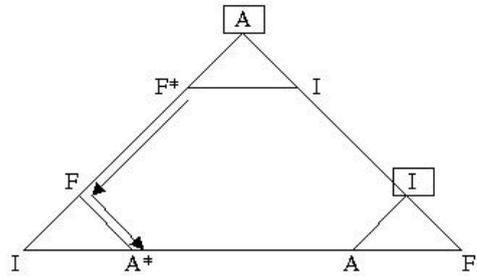
$I_2I_1, I_2A_1, I_2F_1, A_2I_1, A_2A_1, A_2F_1, F_2I_1, F_2A_1, y F_2F_1.$



The corresponding tree diagram shows these 9 positions, represented by each of its possible ways.

¹To begin with the simplest is, almost always, the best beginning.

Now we represent the 9 positions (by a point) and the cases in which it is possible, according to the rules, to go from one position to another in a movement (by a line that joins both contiguous points). Thus, we will have the graph of the game, where the different possible positions of the 2 disks in the 3 towers and the ways that allow to go from one to another position are represented.



For its interpretation you will have to use these keys:

~~✗~~ Each vertex represents a position of the 2 disks in the 3 towers, according to this interpretation: the vertex of the biggest triangle closer (or the same vertex, if it is a vertex of the biggest triangle) shows us the tower where the biggest one is placed (1); the vertex of the smallest triangle shows us the tower where the smallest disk is placed (2); thus:

~~✗~~ The indicated position A corresponds to A_2A_1 : It is the vertex A of the smallest triangle: so, the smallest disk (2) is in tower A. It is the vertex A of the biggest triangle; so, the biggest disk (1) is in tower A.

~~✗~~ The indicated position I corresponds to I_2F_1 : It is the

vertex I of the smallest triangle: so, the smallest disk (2) is in tower I. The vertex of the biggest triangle closer is F; so, the biggest disk (1) is in tower F.

✂✂ If we want to go from the position F^* to the position A^* the shortest way is the one indicated in the graph:

$$F_2A_1 \quad ? \quad F_2I_1 \quad ? \quad A_2I_1$$

The lower vertices of the biggest triangle represent the initial (I2I1) and final (F2F1) positions of the game with 2 disks. It can be easily seen that the shortest way that goes from one to another corresponds to the lower side of the triangle and it shows the minimum necessary movements (3):

$$I_2I_1 \quad ? \quad A_2I_1 \quad ? \quad A_2F_1 \quad ? \quad F_2F_1$$

Let's analyze now the game with three disks:

The possible positions now are 27:

$$I_3I_2I_1, I_3I_2A_1, I_3I_2F_1, I_3A_2I_1, I_3A_2A_1, \dots, F_3F_2F_1$$

In the corresponding graph, we will distinguish three triangles (the biggest, the intermediate and the smallest). Their interpretation is analogous to the one described for the graph of the game with 2 disks: the vertex of the biggest triangle closer (or the same vertex, if it is a vertex of the biggest triangle) shows us the tower where the biggest disk is placed (1); the

vertex of the intermediate triangle closer (or the same vertex, if it is a vertex of the intermediate triangle) shows us the tower where the disk of intermediate size is placed (2); the vertex of the smallest triangle shows us the tower where the smallest disk is placed (3).

 **Find out** what positions of the 3 disks correspond to the vertices marked as 1 and 2. **Look for** in the graph the shortest way to go from position 1 to 2. Finally, **make** the corresponding movements with your game.

 **Locate** and find in the graph the longest way that goes from the initial position of the game with 3 disks (all the disks in the initial tower) to the end (all the disks in the final tower), without repeating any position. **How many** movements do you have to do?

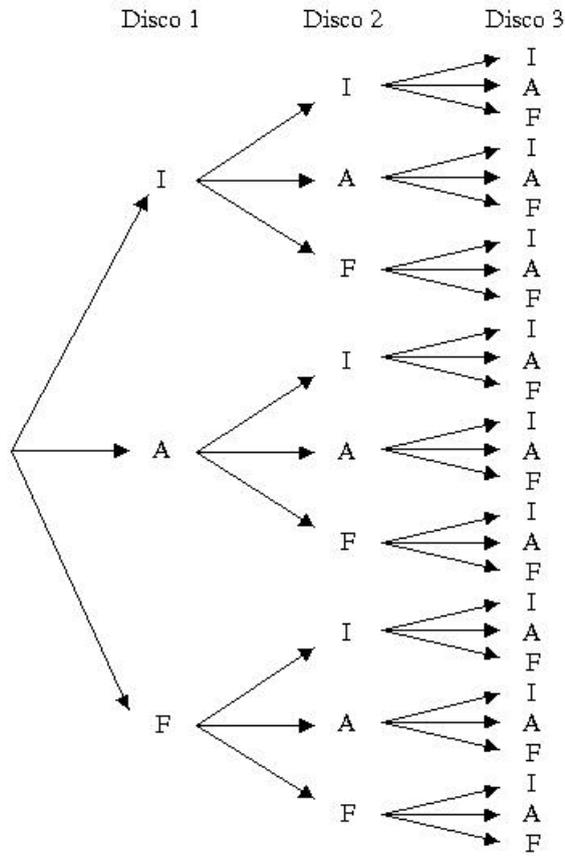
 **Build** the corresponding graph to the game with 4 disks. **Find** the shortest and the longest way, from the initial to the final position of the game, without repeating any position.

YOU WILL NEED:

A pencil, a blank sheet, the game with four disks and the sheet

with the diagram and graph of 3 disks.

Tree Diagram for 3 disks



Graph for 3 disks

