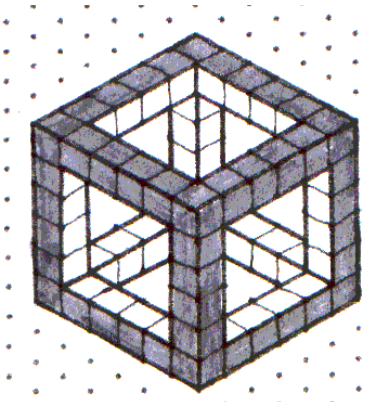
FORMULAE TELL THEIR STORY-2

In the previous activity you could see that the analysis of the formulae has allowed us to know the stories they told us, stories, in that case, about areas.

Now you are going **to count the necessary number of small cubes** to construct the **'frame' of a cube** (a hollow cube: we will only use the necessary small cubes to construct its edges) that has **n** cubes of edge. In the following figure you can see the frame of a cube that has 6 cubes of edge.



You will surely agree that counting the cubes **one by one** is not **an intelligent solution at all**; besides, by doing so, we could only count the cubes of the frame of a certain cube (of 6, 4, 7... cubes of edge) and what we try to do is to get a **formula**, that is to say, **to count the cubes of the frame of a cube that has n cubes of edge**. With it we will be able to easily calculate the number of cubes of the frame of any cube, just by replacing n in the formula by the specific number of cubes of edge that it has.

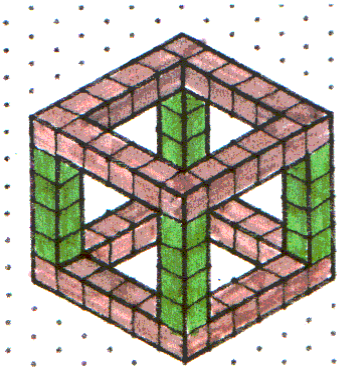
We can get the formula from **as many different ways as there are ways of counting intelligently** the cubes of the frame: dividing into parts, completing and subtracting, etc. For instance:

$$N = 2[n^2 + n + (n-2)^2] + 4(n-2)$$

This comes from dividing its frame into **two parallel 'bases' and four bars or 'posts'** that join them. At the same time, the cubes in the bases have

been counted **completing a square with a side n** and **subtracting** the added square, that is to say, **a squared 'cover' with a side n-2**.

In the following figure, colors help us to see the division we have made to count:

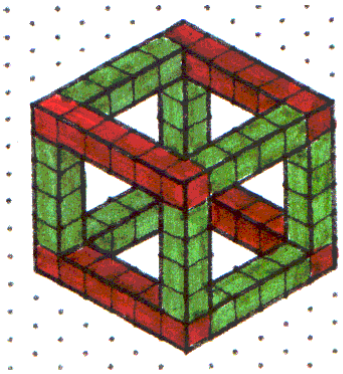


Let's consider another example:

$$N = 4n + 8(n - 2)$$

It has been obtained by **dividing** the frame into four edges of length n and other eight bars of length n-2.

In the following figure we can see this way of dividing the frame:

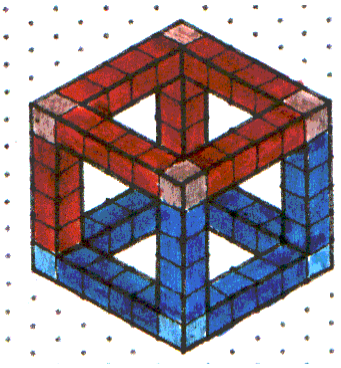


Let's see another example:

$$N = 2[6(n - 2) + 4]$$

It has been obtained by **dividing** the frame into two parts formed by a base and two bars. In order to measure the cubes in each part, it has been **divided into** 6 bars and 4 cubes, placed in the vertices of the base

In the following figure we can see the division of the frame that we have made in order to count in this third case:



✍ **Could you explain how these formulae have been obtained?**

1) $N = 12(n - 2) - 8$

2) $N = 4[n - 2(n - 2)]$

3) $N = n^3 - (n - 2)^3 - 6(n - 2)^2$

Try to obtain **a different formula** from previous ones and represent it by means of cubes of different colours.

YOU WILL NEED:

Fitting small cubes of different colors, pencil and a blank sheet.